

# Classification Calibration Dimension for General Multiclass Losses

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# Background

## Related work

- Binary classification  
*Bartlett et al. 2006.*
- Multiclass 0-1 loss  
*Zhang 2004, Tewari & Bartlett 2007.*
- Hamming loss  
*Gao et al. 2011.*
- Pairwise subset ranking  
*Duchi et al. 2010.*
- Listwise ranking  
*Cossock & Zhang 2008, Xia et al. 2008.*

## Ordinal regression loss

	★	★★	★★★
★	0	1	2
★★	1	0	1
★★★	2	1	0

## Multiclass 0-1 loss

	🤝	⚽	💰
🤝	0	1	1
⚽	1	0	1
💰	1	1	0

## Abstain loss

	🤝	⚽	💰	?
🤝	0	1	1	$\frac{1}{2}$
⚽	1	0	1	$\frac{1}{2}$
💰	1	1	0	$\frac{1}{2}$

# Classification Calibration Dimension

## General loss matrix

	1	2	...	k
1	$L_{1,1}$	$L_{1,2}$	...	$L_{1,k}$
2	$L_{2,1}$	$L_{2,2}$	...	$L_{2,k}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	$L_{n,1}$	$L_{n,2}$	...	$L_{n,k}$


## Definition (Classification calibration dimension)

$$\text{CCdim}(\mathbf{L}) = \min \left\{ d \in \mathbb{N} : \exists \text{ a convex surrogate } \psi : \mathbb{R}^d \rightarrow \mathbb{R}_+^n \text{ that is classification calibrated w.r.t. } \mathbf{L} \right\}$$




## Theorem (Upper bound)

$$\text{CCdim}(\mathbf{L}) \leq \text{rank}(\mathbf{L})$$

# Analysis of Pairwise Subset Ranking Using the Classification Calibration Dimension



The diagram shows a graph  $G$  with nodes and edges. The nodes are arranged in a grid-like structure. The edges are represented by arrows between nodes. The graph is divided into two parts by a vertical line. The left part shows the graph structure, and the right part shows the equation for the pairwise subset ranking loss.

	$\sigma_1$	$\sigma_2$	$\dots$	$\sigma_r!$
				
				
$\vdots$				
				

$$L_{G,\sigma}^{\text{pair}} = \sum_{(i,j) \in E(G)} \mathbf{1}(\sigma(i) > \sigma(j))$$

Lower bound application

For  $r > 4$ ,  $\text{CCdim}(\mathbf{L}^{\text{pair}}) > r$

Poster W70  
Today