Classification Calibration Dimension for General Multiclass Losses

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Background

Related work

- Binary classification
  Bartlett et al. 2006.
- Multiclass 0-1 loss
  Zhang 2004, Tewari & Bartlett 2007.
- Hamming loss
  Gao et al. 2011.
- Pairwise subset ranking
  Duchi et al. 2010.
- Listwise ranking
  Cossock & Zhang 2008, Xia et al. 2008.

Ordinal regression loss

	0	1	2
	1	0	1
	2	1	0

Multiclass 0-1 loss

			$
	0	1	1
	1	0	1
$	1	1	0

Abstain loss

			$	?
	0	1	1	$\frac{1}{2}$
	1	0	1	$\frac{1}{2}$
$	1	1	0	$\frac{1}{2}$

Classification Calibration Dimension

General loss matrix

	1	2	...	k
1	$L_{1,1}$	$L_{1,2}$	$\dots$	$L_{1,k}$
2	$L_{2,1}$	$L_{2,2}$	$\dots$	$L_{2,k}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
n	$L_{n,1}$	$L_{n,2}$	$\dots$	$L_{n,k}$

Definition (Classification calibration dimension)

$$\mathrm{CCdim}(\mathbf{L}) = \min\left\{d \in \mathbb{N} : \exists \text{ a convex surrogate } \psi : \mathbb{R}^d \to \mathbb{R}_+^n \text{ that is classification calibrated w.r.t. } \mathbf{L}\right\}$$

Theorem (Upper bound)

$$\mathrm{CCdim}(\mathbf{L}) \leq \mathrm{rank}(\mathbf{L})$$

Analysis of Pairwise Subset Ranking Using the Classification Calibration Dimension

	σ_1	σ_2	$\dots$	$\sigma_{r!}$
				
				
$\vdots$	$L^{\text{pair}}_{G,\sigma} = \sum_{(i,j)\in E(G)} \mathbf{1}(\sigma(i) > \sigma(j))$			
				

Lower bound application

For $r > 4$, $\quad \mathrm{CCdim}(\mathbf{L}^{\text{pair}}) > r$

Poster W70
Today