E0 232 Probability and Statistics *Due:* Sept 20, 2012

Assignment 3: Answers

1. (a) See solution to Problem 14 in BT Chapter 3.
   (b) See page 153 of BT Chapter 3.
2. (a) 0.5
   (b) 0.1498
   (c) 0.8944
3. $\frac{2-e^{-\lambda t}}{\lambda}$
4. (a) $\frac{15}{32}$
   (b) $f_X(x) = \begin{cases} \frac{15}{32} \times x^2(2 - \frac{x^2}{2}) & \text{if } x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$
   $f_Y(y) = \begin{cases} \frac{15}{32} \times \frac{y^4}{3} & \text{if } y \in [0,2] \\ 0 & \text{otherwise} \end{cases}$
   (c) $f_{X|Y}(x|y) = \begin{cases} \frac{3x^2}{y^3} & \text{if } 0 \le x \le y \\ 0 & \text{otherwise} \end{cases}$. It is defined only when $y \in (0,2]$.
   $f_{Y|X}(y|x) = \begin{cases} \frac{2y}{4-x^2} & \text{if } x \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$. It is defined only when $x \in (0,2)$.
   (d) $E[X] = \frac{5}{4}$, $Var[X] = \frac{17}{1^{12}}$
   $E[Y] = \frac{5}{3}$, $Var[Y] = \frac{5}{63}$
   $Cov(X,Y) = \frac{5}{84}$
   (e) $\frac{47}{64}$
5. $T \sim \text{Exp}(\lambda+\mu)$: $f_T(t) = \begin{cases} (\lambda+\mu)\, e^{-(\lambda+\mu)t} & \text{if } t \ge 0 \\ 0 & \text{otherwise.} \end{cases}$
6. $f_X(x) = \begin{cases} \lambda^2 x e^{-\lambda x} & \text{if } x \ge 0 \\ 0 & \text{otherwise.} \end{cases}$
7. (a) $f_W(w) = \begin{cases} \frac{1}{4\sqrt{w}} & \text{if } 0 \le w < 4 \\ 0 & \text{otherwise} \end{cases}$.
   (b) $Cov(W,X) = 0$
   (c) $f_Z(z) = \begin{cases} \frac{z+5}{16} & \text{if } -5 \le z < -3 \\ \frac{1}{8} & \text{if } -3 \le z < 3 \\ \frac{-z+5}{16} & \text{if } 3 \le z < 5 \\ 0 & \text{otherwise} \end{cases}$
   (d) $Cov(Z,X) = \frac{4}{3}$