E0 232 Probability and Statistics *Due:* Oct 9, 2012

Assignment 4: Answers

1. (a) $0.3e^{-3s} + 0.2 + 0.5e^{5s}$
   (b) $\frac{2e^s(s-1)+2}{s^2}$
   (c) $(\frac{1}{2} + \frac{1}{2}e^s)^{100}(\frac{1}{4} + \frac{3}{4}e^s)^{75}$

2. (a) MGF $= \left(\frac{1}{2} + \frac{1}{2}\frac{e^{10s}-e^s}{9s}\right)^{20}$, Expectation = 55 and Variance = 218.75.
   (b) MGF $= \frac{1}{4}\sum_{k\in\{12,16,20,24\}}(\frac{1}{2} + \frac{1}{2}e^s)^{20k}$, Expectation = 180 and Variance = 2090.
   (c) MGF $= \frac{1}{4}\sum_{k\in\{12,16,20,24\}}\left(\frac{1}{2} + \frac{1}{2}\frac{e^{10s}-e^s}{9s}\right)^{20k}$, Expectation = 990 and Variance = 64437.5.

3. (a) PMF : $p_X(x) = \frac{\exp(\frac{200}{16})(\frac{200}{16})^x}{x!}$
   (b) $\frac{200}{16}$
   (c) $16(1 - e^{-12.5})$

4. (a) Y_n converges to 6.
   (b) Z_n converges to 1.
   (c) U_n converges to 3.5.
   (d) V_n does not converge in probability to any number.

5. (a) i. 0.4520
      ii. -0.6375
      iii. 0.4520
   (b) i. 0.6835
      ii. -0.6375
      iii. 0.6834
   (c) i. 0.9967
      ii. -0.6375
      iii. 0.9975
      iv. 0.9965
   (d) i. 1.0
      ii. -0.6375
      iii. 1.0
      iv. 1.0
   (e) i. 0.2362
      ii. -1.0
      iii. 0.2363
      iv. 0.2361

6. (a) $\frac{\ln(200)}{0.005} = 1060$
   (b) $2\Phi(\frac{5}{\sqrt{2}}) - 1$

7. An example: Random variable $X = \begin{cases} \mu - \epsilon & \text{with probability } \delta \\ \mu & \text{with probability } 1 - 2\delta \\ \mu + \epsilon & \text{with probability } \delta \end{cases}$.

   $\delta = \frac{\sigma^2}{2\epsilon^2}$.

8. $E[X_n] = \sum_{i=1}^{n} \frac{1}{p^i}$